

A Survey on S-Transform

Aditya A. Lokhande

Department of Electrical Engineering, K. K. W. I. E.E & R. Savitribai Phule University of Pune, India

Abstract: *S-transform, which is a powerful time frequency analysis method, has found applications in diverse areas of science and technology. S-transform is a time-frequency spectral localization technique that combines elements of wavelet transform (WT) and Short-Time Fourier Transform (STFT). It is a generalized form of the Gabor transformation where the width of the Gaussian window scales inversely and the height of the window scales linearly with frequency. In this Paper We Will get familiar with general S-Transform and its Inverse S-Transform. Then will see some applications like Power Frequency Measurement, Signal Filtering, Power System Disturbance Recognition, Fault Detection and Diagnosis of Grid-Connected Power Inverters, Geophysical Signal Analysis, EEG analysis, Optical 3-D Surface Profile Measurement, Magnetic Resonance Imaging (MRI).Traditionally S-transform is use to measure time-frequency so will compare S-transform with Gabor Transform, Wigner Transform, short-time Fourier transform.*

Keywords: *S-Transform, Wavelet Transform (WT), Short-Time Fourier Transform (STFT), Gabor Transform, Wigner Transform.*

I. Introduction

Now a day's non-stationary time series such as speech, electrocardiograms, seismic signal and genomic signal are gaining more importance as they contain time-bounded events and artifacts. In depth study of such events requires the determination of time-local spectra. During the last few years many methods of determining local spectra have been proposed. These include time-scale approach as in the continuous wavelet transform (CWT) and time-frequency approach as in the short-time Fourier transform (STFT). The main disadvantages of the STFT are its inability to detect and resolve low frequencies, and poor time resolution of high frequency events, both these problems are due to the fixed width of the STFT window. Improved performance is observed by the wavelet transform, but it produces time-scale plots that are unsuitable for intuitive visual analysis. Also the absolute referenced phase information cannot be deduced from the wavelet transform. A superior method for time-frequency analysis known as S- transform has been evolved which enjoys the advantages of both STFT and wavelet transform. The basic idea is to obtain a time -frequency energy distribution of the signal so that we can isolate and process independently the components of the signal in the time-frequency plane. The S-transform, introduced by Stockwell, Mansinha, and Lowe, suitably combines the strengths of the STFT and wavelet transforms. It is the hybrid of short-time Fourier and Wavelet analyses. It employs a scalable and variable window length and uses the Fourier kernel to provide the phase information referenced to the time origin. Hence, it provides supplementary information about spectra which is not available from locally referenced phase obtained by the continuous wavelet transform. The S-transform has found extensive applications in many fields such as Geophysics [6] , Biomedical Engineering [9],Power transformer protection [5], Power quality analysis [3], oceanography , atmospheric physics , medicine, hydrogeology and mechanical engineering.

The S-transform (ST) can be derived either as an short-time Fourier transform (STFT) where the window width depends on the frequency, or as a phase correction to the wavelet transform (WT)[8]. The S transform has particular advantages compared with the STFT and the WT. The ST uses sinusoidal basis functions like those of the FT and STFT. These are multiplied by Gaussian window functions that vary in width depending on the frequency under consideration, unlike the constant width STFT windows. Consequently, the ST provides both the true frequency and globally referenced phase of the FT and STFT and the progressive resolution of the WT[8]. The resolution of the ST depends on the frequencies, and the basic wavelet need not satisfying the admissible condition. The reciprocal of the frequency is used as the scaling factor in ST, since that we can get the high resolution of the frequencies domain in low frequencies with a wider time window, and the high resolution of the time domain in high frequencies with a narrower time window. The another important character is that the ST is a lossless method and the inverse of ST is the Fourier transform, since that it relates to the Fourier spectrum closely. That means the ST will not lose any valuable information, and it can reversed back easily. These properties have made ST useful in a variety of image/signal analysis tasks. As a super excellent space-frequency methods, image reconstruction methods and a close contact with the FT, ST suits for medical image processing greatly

II. S-Transform

The Fourier transform lacks the skill to position time and frequency at the same time. It is not available for the time frequency localization. Gabor first proposed that it could adopt a moving and scalable localizing Gaussian window as the base function. It defined by

$$w_{r,\omega}(t) = w(t-r)e^{i\omega t} \quad \dots (2.1)$$

Short-time Fourier transform is
$$STFT(r, f) = \int_{-\infty}^{\infty} x(t)w(t-r)e^{-j2\pi ft} dt \quad \dots (2.2)$$

According to the uncertainty principle, the time-bandwidth product could not get smaller without limits. The Gaussian window is able to combine the time domain and frequency domain. The Gaussian window is defined

as
$$\omega(t) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{t^2}{2\delta^2}} \quad \dots (2.3)$$

According to the nature of Gaussian window, δ is the scale factor to change the width of Gaussian window. In order to make the width of Gaussian window have a better self adaptability to different frequency components. δ Could be defined as a frequency-related function

$$\delta(f) = \frac{1}{|f|} \quad \dots (2.4)$$

The new function is,
$$\omega(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} \quad \dots (2.5)$$

We can get the expression of S-transform

$$ST(r, f) = \int_{-\infty}^{\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(r-t)^2}{2}} e^{-i2\pi ft} dt \quad \dots (2.6)$$

In which f is the frequency, t and r the time variables.

III. Inverse S-Transform

The S-transform is a representation of local spectrum. It could expect a simple operation of averaging the local spectra over time to give the Fourier spectrum.

$$x(f) = \int_{-\infty}^{\infty} S(r, t) dt \quad \dots (3.1)$$

The Fourier transform of $x(t)$ is

$$x(t) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad \dots (3.2)$$

The relationship between Fourier transform and S-transform is

$$x(t) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} S(r, t) dt \quad \dots (3.3)$$

The signal $x(t)$ could exactly recoverable from S-transform, so the Inverse S-Transform Will be

$$x(t) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} S(r, t) dt \right\} e^{j2\pi ft} dt \quad \dots (3.4)$$

IV. Application

- **Power Frequency Measurement [1]**

The power frequency is an important operation parameter of power systems. It is widely used in generator protection, synchronism checking, load shedding frequency protection relays, and volts per Hertz relay. But fast and accurate frequency estimation in the presence of harmonic distortion and noisy signal is a challenging problem.

S-Transform can accurately characterize all the harmonic components of a signal. Because it has good frequency resolution in low frequency components, and a high time distinction power in high frequency area.

- **Signal Filtering [2]**

In signal processing, a filter is a device or process that removes some unwanted components or features from a signal. Filtering is a class of signal processing, the defining feature of filters being the complete or partial suppression of some aspect of the signal. Most often, this means removing some frequencies and not others in order to suppress interfering signals and reduce background noise. However, filters do not exclusively act in the frequency domain; especially in the field of image processing many other targets for filtering exist. Correlations can be removed for certain frequency components and not for others without having to act in the frequency domain.

- **Power System Disturbance Recognition [3-4]**

The power quality supplies have been a major concern of electricity users these days. The definition of a power quality problem is that any variation in the electrical power service, such as voltage dips, voltage swell, momentary interruptions, harmonics, transients, notches and noises which results in disoperation or failure of end-use equipment. The voltage and current waveform are distorted due to presence of harmonics in it which in turn creates many problems.

- S transform has been proven to be able to identify a few types of disturbances, like voltage sag, voltage swell, momentary interruption, and oscillatory transients.
- S transform also be applied for other types of disturbances such as notches, harmonics with sag and swells etc.
- S transform generates contours which are suitable for simple visual inspection. However, wavelet transform requires specific tools like standard multi resolution analysis.

- **Fault Detection and Diagnosis of Grid-Connected Power Inverters [5]**

With the continuing need of productivity increase and better performance specifications lead to more demanding operating condition of many electrical systems. Among these systems are three-phase voltage source power inverters, which cover a wide range of applications from adjustable-speed drives to renewable energy grid connections. Usually a fault in the power inverter forces a stop in the associated system for a non-programmed maintenance procedure. Thus preventive maintenance programs are essential to enhance the overall system productivity. However, in order to implement preventive maintenance programs is mandatory to develop accurate fault detection procedures. A new approach for the detection of open switch faults, and respective diagnosis of which switch is open, in a voltage source power inverter was presented. This approach is based on the application of the S-Transform on the output currents. The obtained S matrix presents specific patterns that allow the identification of a fault. The time-frequency contour allows the detection and identification of the faulty switch.

- **Geophysical Signal Analysis [6]**

Geophysical signal analysis is concerned with the detection and a subsequent processing of signals. Any signal which is varying conveys valuable information. Hence to understand the information embedded in such signals, we need to 'detect' and 'extract data' from such quantities. Geophysical signals are of extreme importance to us as they are information bearing signals which carry data related to petroleum deposits beneath the surface and seismic data. Analysis of geophysical signals also offers us a qualitative insight into the possibility of occurrence of a natural calamity such as earthquakes or volcanic eruptions. Gravitational and magnetic fields are detected using extremely sensitive gradiometers and magnetometers respectively. The gravitational field changes are measured using devices such as atom interferometers. A superconducting quantum interference device (SQUID) is an extremely sensitive device which measures minute changes in the magnetic field. After detection, the data from these signals is extracted by performing spectral analysis, filtering and beam forming techniques. These techniques can be used in oil exploration to estimate the position of underground objects, harnessing energy. SQUID is based on S-Transform.

- **EEG analysis [11]**

EEG (Electroencephalogram) signal is the electrical activity of the brain, which is recorded by attaching electrodes to the scalp. The brain is much complicated in structure, and contains abundant information related to the human spirit and biological structures. Therefore, many researchers from kinds of fields unceasingly extract and analyze the implicit information of EEG by all sorts of signal processing techniques. At present, the main methods in EEG analysis include time domain analysis, frequency domain analysis, time-

frequency analysis, artificial neural network analysis, nonlinear dynamics analysis, independent component analysis, etc and the time-frequency analysis is a type of important method, by which the one-dimensional signal is mapped to two-dimensional plane to fully reveal the time-frequency characteristics of EEG. It can not only reflect the frequency distribution but also show the duration of every frequency component. Major time-frequency methods S-transform develops on the basis of STFT and CWT (Continuous Wavelet Transform) as a time frequency analysis technique, enjoying many advantages, such as linearity, loss less reversibility, multi-resolution, good time-frequency resolution, etc.

- **Optical 3-D Surface Profile Measurement [12]**

The S transform is an extension of the idea of the continuous wavelet transform (CWT) and is based on a moving and scalable localizing Gaussian window. It has been applied to many disciplines, such as cardiology, audiology, seismology because of this advantages now it used to measure optical 3D surface profile. First We obtained the fringe patterns from Fringe Projector setup with Michelson Interferometer. After that, these patterns were evaluated by using S-faz algorithm and Continuous Wavelet-Phase algorithm. As a result, it was seen that the S-phase method gave a better 3-D profile than other method and showed the high frequency resolution.

- **Magnetic Resonance Imaging (MRI) [13]**

The one-dimensional ST, applied to a signal (such as time Course MRI data), can be used to localize and remove noise components and artefacts. This technique has proven useful for filtering and artefact removal in MR imaging. The two dimensional ST provides the local text & information for each point in an image. This information can be used to distinguish between tissues of differing appearance. Texture may be an indicator of disease activity – for example, a texture map of an MR image may enhance lesions or other abnormalities that are difficult to distinguish in conventional MR images.

V. Comparison With Other Time-Frequency Analysis Tools

1) Comparison with Gabor Transform

The only difference between Gabor Transform (GT) and S Transform is the window size. For GT, the windows size is a Gaussian function; meanwhile, the window function for S-Transform is a function of f . With a window function proportional to frequency, S Transform performs well in frequency domain analysis when the input frequency is low. When the input frequency is high, S-Transform has a better clarity in the time domain. As table below.

Frequency	Gabor Transform	S-Transform
Low-frequency	Bad clarity in time domain	Good clarity in frequency domain
High-frequency	Bad clarity in frequency domain	Good clarity in time domain

This kind of property makes S-Transform a powerful tool to analyze sound because human is sensitive to low frequency part in a sound signal.

2) Comparison with Wigner Transform

The main problem of Wigner Transform is the cross term which caused by the auto-correlation function in the Wigner Transform function. Cross term may cause noise and distortion to signal analysis. No cross term may appear by using the S-transform analysis which makes S Transform a suitable tool to design filter or signal modulation.

3) Comparison with the short-time Fourier transform

We can compare the S transform and short-time Fourier transform (STFT).[2][6] First, a high frequency signal, a low frequency signal, and a high frequency burst signal are used in the experiment to compare the performance. The S transform characteristic of frequency dependent resolution allows the detection of the high frequency burst. On the other hand, as the STFT consists of a constant window width, it leads to the result having poorer definition. In the second experiment, two more high frequency bursts are added to crossed chirps. In the result, all four frequencies were detected by the S transform. On the other hand, the two high frequencies bursts are not detected by STFT. The high frequencies bursts cross term caused STFT to have a single frequency at lower frequency.

VI. Conclusions

In this research, it is found that The S-transform is based on a moving and scalable localizing Gaussian window. It has some desirable characteristics that are absent in the continuous wavelet transform, and it is unique in that it provides frequency dependent resolution while maintaining a direct relationship with the Fourier spectrum. The wavelet does not satisfy the condition of zero mean for an admissible wavelet; therefore, it is not strictly a continuous Wavelet transform.

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